Home Work I: Lecture 1-4; due by Thursday March 6, 2008

1. For a log-normal aerosol size distribution, geometric properties can be defined from the geometric mean diameter using the relationship, $\overline{D}_{pv} = \overline{D}_{pg} \exp(v \ln^2 \sigma_g)$, where v is a parameter that defines the particular mean diameter of interest and σ_g is the standard deviation of the distribution.
    For v =0, 1, 2, 2.5, and 3, what is the name of the corresponding diameters.

υ	name
0	Geometric mean
1	Surface area mean
2	Surface area median
2.5	Effective diameter
3	Volume median

2. Using the following table of segregated aerosol size, try to fit, one for each mode, two log-normal distributions by providing for both the geometric mean diameter and geometric standard deviation. Then calculate the volume mean diameter of the corresponding lognormal distribution of the volume size for the two modes.

Size range (diameter, μm)	Concentration (cm^{-3})	Cumulative concentration (cm^{-3})	Normalized concentration ($\mu m^{-1} cm^{-3}$)
0.001 - 0.01	100	100	11111
0.01-0.02	200	300	20000
0.02-0.03	30	330	3000
0.03-0.04	20	350	2000
0.04-0.08	40	390	1000
0.08-0.16	60	450	750
0.16-0.32	200	650	1250
0.32-0.64	180	830	563
0.64-1.25	60	890	98
1.25-2.5	20	910	16
2.5-5.0	5	915	2
5.0-10.0	1	916	0.2

ANSWER

$$D_{g1} = \exp\left[\frac{\sum_{i=1}^{4} N_i \ln(\sqrt{D_{i+1}D_i})(\ln(D_{i+1}) - \ln(D_i))}{\sum_{i=1}^{4} N_i (\ln(D_{i+1}) - \ln(D_i))}\right] = 0.0058 \, \mu m$$

$$\sigma_{g1} = \exp\left[\sqrt{\frac{\sum_{i=1}^{4} N_i (\ln(\sqrt{D_{i+1}D_i}) - \ln(D_{g1}))^2 (\ln(D_{i+1}) - \ln(D_i))}{\sum_{i=1}^{4} N_i (\ln(D_{i+1}) - \ln(D_i))}}\right] = 2.2$$

$$D_{g2} = \exp\left[\frac{\sum_{i=5}^{12} N_i \ln(\sqrt{D_{i+1}D_i})(\ln(D_{i+1}) - \ln(D_i))}{\sum_{i=5}^{12} N_i (\ln(D_{i+1}) - \ln(D_i))}\right] = 0.3 \, \mu m$$

$$\sigma_{g2} = \exp\left[\sqrt{\frac{\sum_{i=5}^{12} N_i (\ln(\sqrt{D_{i+1}D_i}) - \ln(D_{g2}))^2 (\ln(D_{i+1}) - \ln(D_i))}{\sum_{i=5}^{12} N_i (\ln(D_{i+1}) - \ln(D_i))}}\right] = 2.3$$

$$D_{V1} = D_{g1} \exp(3\ln^2(\sigma_{g1})) = 0.036 \, \mu m$$

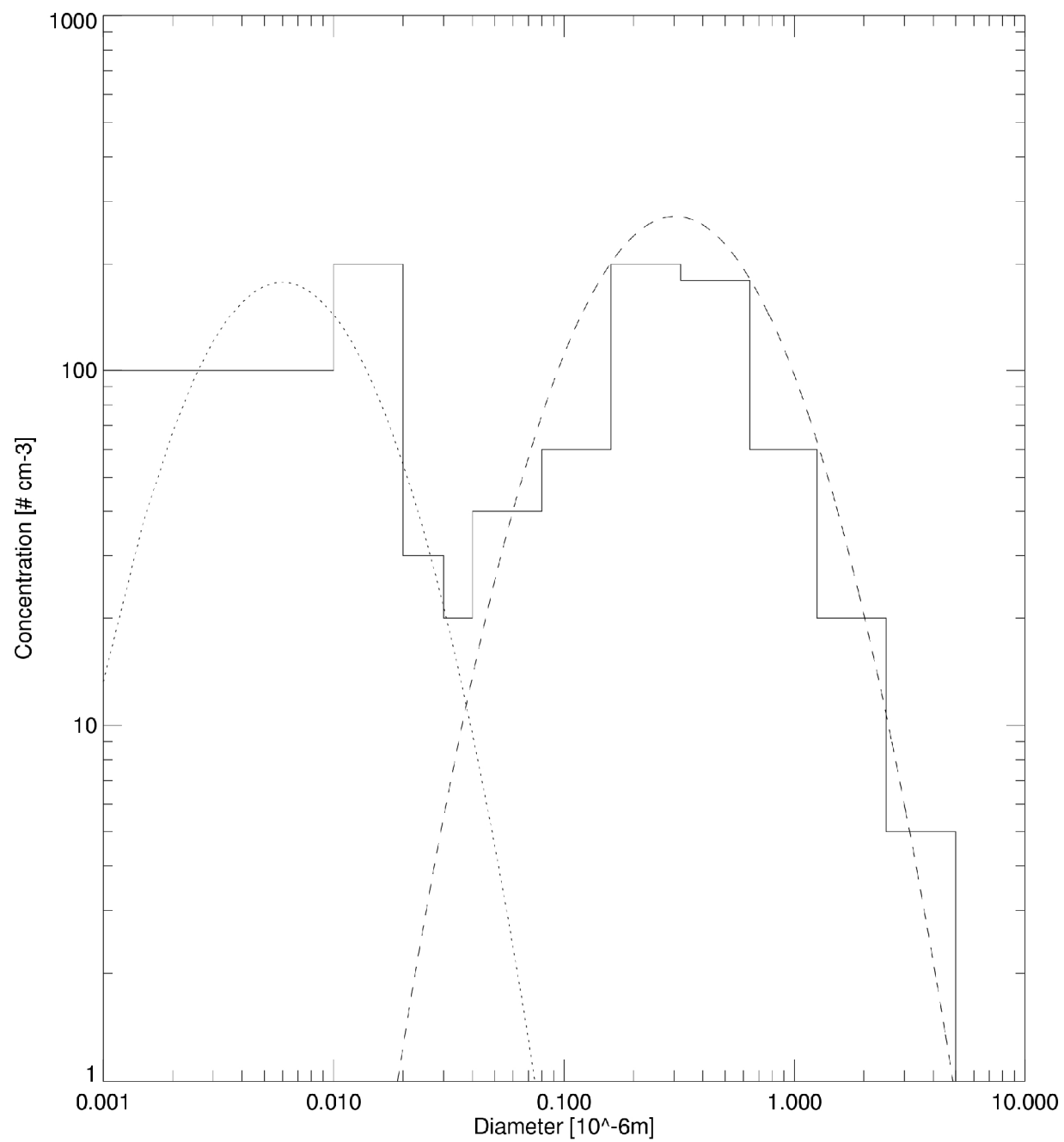$$D_{V2} = D_{g2} \exp(3\ln^2(\sigma_{g2})) = 2.4 \, \mu m$$

Figure 1. fitting of measured size distribution (continuous line) by 2 lognormals (dot and dash lines)

Indicate for the following questions if the correct answer is always (A), sometimes (S), or never (N) true.

    a. A_S_**N**_. The slip correction factor has a value less than 1.

    b. **A**_S_N_. The slip correction factor is higher on top of the Everest than at sea-level.

    c. A_S_**N**_. PM2.5 of the primary standards for particulate matter set by the EPA corresponds to the mass concentration of all particles greater than 2.5 μm.

    d. **A**_S_N_. PM10 of the primary standards for particulate matter set by the EPA corresponds to the mass concentration of all particles less than 10 μm.

    e. A_S_**N**_. Particles in the accumulation mode are finer than in the nuclei mode

    f. **A**_S_N_. Particles in the coarse mode are mechanically emitted into the atmosphere.

    g. **A**_S_N_. The gravitational settling is the major mechanism to limit aerosol radius to about 10 μm

    h. **A**_S_N_. The wet deposition is the major mechanism to remove aerosols in the accumulation mode.

3. The 24-hour measurement of the aerosol optical depth (τ) at 550 nm of a 2 km thick aerosol layer composed homogeneously of dust and dry sulfate (the relative humidity is below the value of deliquescence) gives a time averaged value of 1, while the averaged measurement of absorption optical depth is 0.02. What are the concentration of dust and sulfate? What are the values of PM2.5 and PM10? And does the values are within the EPA standards?
To solve this, we can assume that dust and dry sulfate distributions are log-normal with a geometric mean radius of 0.4 and 0.07 μm, and a standard deviation of 2.2 and 2, respectively. As a good approximation, τ can be estimated from $\tau = \dfrac{3Q_{ext}M_p}{2\rho_p Dg_{eff}}$, where

Q_{ext} is the extinction efficiency at 550nm (2.2 for dust and 1.3 for dry sulfate), ρ_p is the mass density (2.6 g/cm^3 for dust, 1.7 g/cm^3 for dry sulfate), Dg_{eff} is the effective diameter, M_p is the vertically integrated mass column of aerosol type p. We can further assume that the aerosols are externally mixed and that at 550 nm the single scattering albedo of dust is 0.97 and of dry sulfate 1. Recall that D_{geff} can be easily calculated from D_g and σ_g (cf. Exercise 1 above), and the single scattering albedo ω of an external mixture of 2 aerosol types is calculated as $\omega = \dfrac{\sum\limits_{p=1}^{2}\omega_p\tau_p}{\sum\limits_{p=1}^{2}\tau_p}$

ANSWER

DZ=2000 m

RHO1=2600 kg.m^{-3}

RHO2=1700 kg.m^{-3}

Q1=2.2

Q2=1.3

SIG1=2.2

SIG2=2

Dust effective diameter DE1=3.8 μm

Sulfate effective diameter DE2=0.47 μm

Dust volume median diameter DV1=5.2 μm

Sulfate volume median diameter DV2=0.6 μm

SSA1=0.97: single scattering albedo of dust

TAU1: dust optical depth

SSA2=1: sulfate single scattering albedo

TAU2: sulfate optical depth

TAU=TAU1+TAU2= 1

SSA=(SSA1*TAU1+SSA2*TAU2)/TAU

TAUA=(1-SSA)*TAU

SSA=1.-TAUA/TAU = 0.98

TAU1=TAU*(SSA-SSA2)/(SSA1-SSA2) = 0.667

TAU2=TAU-TAU1 = 0.333

M1=TAU1*2/RHO1*DE1/3/Q1 = 2 g.m^{-2}

M2=TAU2*2/RHO2*DE2/3/Q2 = 0.14 g.m^{-2}

Dust concentration = M1/DZ = 994 μg.m^{-3}

Sulfate concentration= M2/DZ =68 μg.m^{-3}

The mass distribution, represented by a log-normal, is integrated up to 2.5 μm diameter for both dust and sulfate to obtain PM2.5. For PM10, the log-normal of dust and sulfate are integrated up to 10 μm diameter.

PM2.5=66.5 μg.m^{-3} which is above the 35 μg.m^{-3} maximum concentration over 24 hours set by the EPA

PM10=862 μg.m^{-3} which is above the 150 μg.m^{-3} maximum concentration over 24 hours set by the EPA

PM2.5 by dust =0.18 μg.m^{-3} which represents 18% of dust mass

PM2.5 by sulfate=66.32 μg.m^{-3} which represents 98% of sulfate mass

PM10 by dust = 794 μg.m^{-3} which represents 80% of dust mass

PM10 by sulfate = 67.6 μg.m^{-3} which represents more than 99.99% of sulfate mass